



# Proceedings Report

## Finding Synergies in the Mathematical Sciences



science  
& technology

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The Academy of Science of South Africa (ASSAf) was inaugurated in May 1996. It was formed in response to the need for an Academy of Science consonant with the dawn of democracy in South Africa: activist in its mission of using science and scholarship for the benefit of society, with a mandate encompassing all scholarly disciplines that use an open-minded and evidence-based approach to build knowledge. ASSAf thus adopted in its name the term 'science' in the singular as reflecting a common way of enquiring rather than an aggregation of different disciplines. Its Members are elected on the basis of a combination of two principal criteria, academic excellence and significant contributions to society.

The Parliament of South Africa passed the Academy of Science of South Africa Act (*Act 67 of 2001*), which came into force on 15 May 2002. This made ASSAf the only academy of science in South Africa officially recognised by government and representing the country in the international community of science academies and elsewhere.

This report reflects the proceedings of the Finding Synergies in the Mathematical Sciences Workshop held on 15 – 16 September 2016 at the Hotel Verde, Cape Town, South Africa. Views expressed are those of the individuals and not necessarily those of the Academy nor a consensus view of the Academy based on an in-depth evidence-based study.

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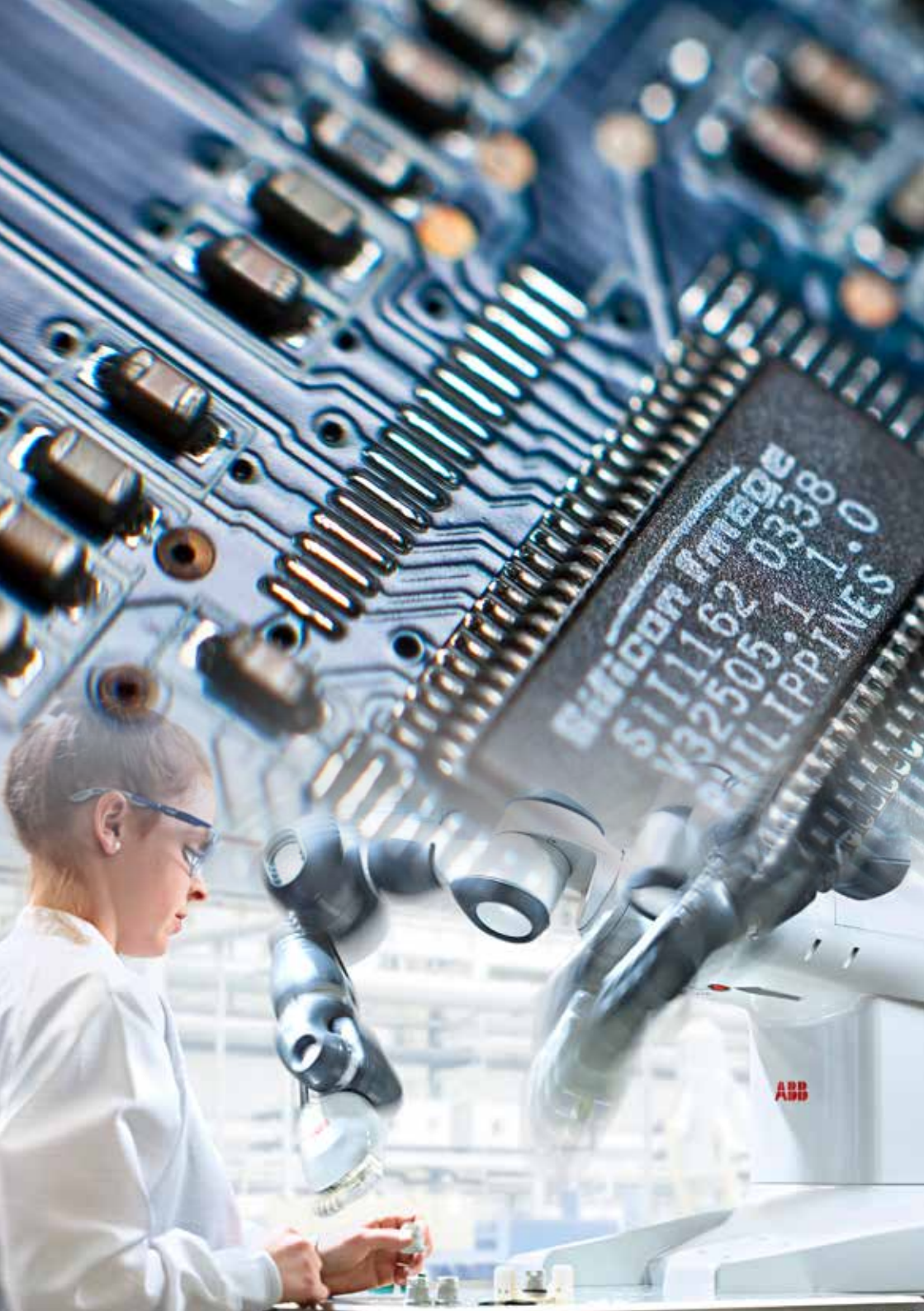
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Professor Fritz Hahne  
*Chair: STEM Education Standing Committee*

## ABBREVIATIONS/ACRONYMS

<b>AIMS</b>	African Institute for Mathematical Sciences
<b>ASSAf</b>	Academy of Science of South Africa
<b>CERN</b>	European Organisation for Nuclear Research
<b>IAU</b>	International Astronomical Union
<b>NDP</b>	National Development Plan
<b>NRF</b>	National Research Foundation
<b>PGCE</b>	Postgraduate Certificate in Education
<b>SDGs</b>	Sustainable Development Goals
<b>SKA</b>	Square Kilometre Array
<b>STEM</b>	Science, technology, engineering and mathematics
<b>WMCS</b>	Wits Mathematics Connect Secondary





**ABB**

# 1 INTRODUCCION

## 1.1 Background

The Academy of Science of South Africa (ASSAf) convened a workshop on Finding Synergies in the Mathematical Sciences in Cape Town on 15 – 16 September 2016. The meeting sought to promote and seek a discursive space to explore synergies and an inclusive approach to mathematics across all the scientific disciplines and the adoption of a broader, more integrated vision of the role of mathematics in education and society. The meeting also aimed to explore the feasibility and desirability of a broad consensus study which would *inter alia* conduct an audit of initiatives and best practices that address issues of synergising the mathematical sciences in South Africa.

South Africa's National Development Plan projects that the number of school-leavers, who should qualify for entrance to a Bachelor's degree for which mathematics is a prerequisite, will more than double by 2024 and will treble by 2030. At the same time, the relationship, or lack of one, between mathematics and mathematics education represents a matter of concern, potentially exacerbating the limited performance of South African pupils and students.

The workshop sought to address these significant capacity challenges and the increasing permeation of mathematics across academic disciplines by exploring potential synergies and collaborations among the mathematical sciences, with the interconnected goals of improving educational and research outputs and supporting the country's development and transformation.

## 1.2 Mathematics in society

Science, technology, engineering and mathematics degrees and careers based on these qualifications are popular and can have relatively great material benefits for South African students. Parents almost unquestioningly support mathematics as a subject for study. The higher education sector has a responsibility to leverage such social preferences more effectively. Mathematics is a cultural asset and should be widely promoted as such in science communication policies and programmes.

In terms of universities' role in society at large, mathematics and science departments should show that they can respond proactively to shaping the agenda for change in higher education and take a broader look at transformation – moving it beyond discussions about classroom and

assessment practices to address South Africa's broader social developmental needs.

Mathematics should be taught in a welcoming, democratic way that encourages everyone to learn and banishes the notion of the subject as the preserve of an innately predisposed elite. This open culture of mathematics needs to be built from the pre-school level. Role models with a genuine love and enthusiasm for their own subjects may be able to break down some of the barriers.

As an academic subject, the mathematical sciences should not merely be tailored to the demands of business and industry, but should also help to sustain the educational enterprise, producing future researchers and teachers.

### 1.3 Mathematics and the students

The teaching of mathematics at university should eschew an instrumental approach in favour of creating a pedagogic culture that supports a deeper understanding of mathematics among students (who may be scared of the subject) creating familiarity with certain approaches and methodologies.

Moving from mathematical models to real-life situations is the biggest challenge faced by many students. Students should be equipped with a nuanced understanding of the scientific and economic models to which they have been exposed and the disposition to seek out new knowledge and apply their mathematics skills accordingly.

Universities' biggest social impact is in their teaching. The study and use of mathematics raise important issues of disposition, identity and culture among students, who should be encouraged to define their academic and other goals. The pedagogy at university should excite and inspire students in pursuit of these goals.

### 1.4 Mathematics and the sciences

Much leading-edge scientific research is driven by mathematics. Mathematics can provide solutions for the earth, biological, engineering, physical and economic sciences and these disciplines also raise fundamental questions that mathematics could usefully seek to answer. Careful consideration should accordingly be given to the level at which the required mathematics should be taught – this may often be as required at postgraduate level. Alternatively, interdisciplinary majors at under-



graduate level could increasingly be an option or radical changes to curricula may be in order. In general, university departments could usefully review which mathematical tools they should be teaching.

Although mathematicians have to build their own discipline – which in the field of pure mathematics may have applications that have yet to become evident – the artificial divide between pure and applied mathematics is not generally beneficial for students or researchers. Departmental silos within universities are hampering progress towards the mutually beneficial integration of mathematical studies among disciplines. Mathematics departments alone should not bear the brunt of this – other departments should also help to teach the required mathematics.

## 2 INTEGRATING MATHEMATICAL STUDIES

It is questionable whether large-scale changes to curricula would help to integrate mathematical studies more effectively among related disciplines. It may be preferable to revise the examinations themselves – and thus the syllabus – to improve outcomes.

Courses should be adapted to include big data, although the topic may best be addressed at postgraduate level. Big data studies should not be only application-driven and should support the study of mathematics for itself.

Universities should support the creation of a culture in which mathematicians (of whatever specialisation) talk to each other freely, which should filter through into improved curricula and greater cross-disciplinary inputs. More interdisciplinary projects and workshops can provide a way forward with those that are successful providing models for the future.

The research topics chosen and supported in departmental silos are often driven by turf politics, self-promotion, the availability of research grants and the need for National Research Foundation approval. This can militate against the production of important and relevant mathematical work in South Africa. University teaching staff should pay greater attention to international developments in interdisciplinary mathematics and should also update their courses to include more recent developments and discoveries in their fields.

Greater collaboration may also be fostered in mathematics and the sciences by large-scale democratic projects conducted via the internet and social media.

## 2.1 Mathematics in business

There is a lack of articulation between the research conducted by economics and mathematics departments at universities and the models that are required by the financial sector. Closer relations between mathematics and economics departments and the financial sector could address this.

## 2.2 Mathematics at school

The inadequate mathematics skills of many new students at universities can be traced back to how and what they were taught at school. The benefits of the many intervention programmes for in-service teachers that are conducted in South Africa at great expense need to be properly evaluated and collated. The programmes that are found to succeed – those that address teachers' mathematical and pedagogical skills – should be rolled out. The status of mathematics (and other) teachers needs to be raised.

Mathematics departments should encourage students into mathematics teaching as a career and help to equip them with the required skills. This would also encourage pedagogy that constructively addresses the shortfalls in understanding and knowledge among new students.

The extent and kind of integration between education and mathematics/science departments need to be reviewed carefully. Universities may also help to shape mathematics/science inputs into teaching diplomas.

# 3 TOWARDS A NEW MATHEMATICS CULTURE

Although a more open mathematics culture cannot be declared, its creation and change in the current academic culture can be supported through the commitment of key individuals. Greater communication on interdisciplinary concerns should be fostered within and among universities and with relevant non-governmental and governmental bodies.

# 4 WORKSHOP PROCEEDINGS

## 4.1 Introduction

The Academy of Science of South Africa (ASSAf) convened a workshop on Finding Synergies in the Mathematical Sciences in Cape Town on

15 - 16 September 2016. In response to an identified global need, the meeting sought to enable South African mathematical science practitioners to explore synergies and identify areas of mutually beneficial collaboration. To this end, the ASSAf workshop provided a platform for presentations and deliberation by key experts in mathematics; the natural and physical sciences; statistics, economics and data sciences; and the social sciences, including education.

With the advent of democracy and equality in South Africa in 1994, the promotion of access to science and technology became a national agenda. Despite the legacy of insufficient, unequal participation and success in mathematics and the sciences and an overwhelming backlog in the knowledge and experience base of students, great efforts have been made to ensure epistemological access for all. However, the overall performance of students in the mathematical sciences remains limited and, despite widespread interventions, little progress appears to have been made to alleviate this.

South Africa's National Development Plan projects that the number of school-leavers, who should qualify for entrance to a Bachelor's degree for which mathematics is a prerequisite, will more than double by 2024 and will treble by 2030. This poses significant capacity challenges to the mathematical sciences, which should seek to leverage this growth for their own good and to the benefit of the country's development and transformation. In this regard, the relationship, or lack of one, between mathematics and mathematics education also represents a matter of great concern. The two disciplines generally remain discrete and mutually exclusive.

Meanwhile, American and other researchers globally predict that the permeation of mathematics into many disciplines will continue to increase.<sup>1</sup> In this regard, it is important to analyse best practices of approaches that have embraced this trend and consider specific topics and issues – such as the emergence of 'big data' – across all the disciplines that pose particular strategic challenges to the mathematical sciences.

Universities, schools and policymakers all need to take responsibility for addressing these challenges and forge a common vision accordingly. Accordingly, the ASSAf workshop was held to promote an inclusive approach to mathematics across all the scientific disciplines and the adoption of a broader and more integrated vision of the role of

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<sup>1</sup> National Research Council of the National Academies, *The Mathematical Sciences in 2025* (Washington DC, The National Academies Press, 2013).

mathematics in education and society. In order to counter fragmentation within the mathematical sciences, the meeting sought to explore synergies among the disciplines across the academic project and how these may be leveraged for collaborative research and to generate new knowledge.

## 5 PLENARY 1: EARTH AND BIOLOGICAL SCIENCES

Biochemists used to kill living organisms, then separate and study the bits.<sup>2</sup> The analytical method was 'divide and conquer'. Now the approach is becoming one of synthesis – 'integrate and rule'. Systems biologists seek to understand life by considering how biological function emerges from the interactions between components of living systems.

The discipline seeks to formulate theoretical frameworks for the metabolic network, in which reactions are catalysed by enzymes. Computational systems biologists seek to chart the dynamics of the relationships among enzymes, which are like little productive machines, and clusters of protein molecules, which are like little factories.

On one level, mathematics offers a quantitative system for describing what happens in this network. Systems of ordinary non-linear differential equations can describe the topology of the network – what is connected to what – and its dynamics, how fast things can happen. In the graph of a reaction network – in which metabolites may be represented by dots and the reactions among them by connecting lines – each of the hundreds of reactions that are charted proceeds at a rate described by its own rate equation, which must be ascertained experimentally.

Besides ordinary non-linear differential equations, other mathematical tools employed by metric or quantitative biologists include stability and bifurcation analysis, deterministic chaos, partial differential equations, difference equations, cellular automata, stochastic differential equations, hybrid systems and scientific computing.

On another level, questions have emerged that require the adoption of broader mathematical approaches in biology. System biologists seek to understand the functional life of a hugely complex system – the cell. Adequate description of the cell's functional organisation entails analysis of the 'emergent' properties of its system. These properties emerge from the relationships between the components of the system, which can be

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<sup>2</sup> Jannie Hofmeyr, Biology and Mathematics, presentation at ASSAf Mathematical Sciences Workshop, Finding Synergies in the Mathematical Sciences, Cape Town, 15 - 16 September 2016.



described by category theory (which deals with relationships between objects), rather than by set theory (which classifies objects).

Mathematical biology is particularly concerned with the act of modelling. Modelling, which has been described by American theoretical biologist, Robert Rosen, as the essence of science and the basis of epistemology, enables the observer to understand, predict or control phenomena. Modelling may be regarded as fundamental to human understanding. For example, the first thing that people do when they wake up is to model the world according to their perceptions. Science may be regarded as the art of establishing appropriate modelling relations between the natural world and that of mathematical and other formalisms.

In science, effective modelling operates by analogy rather than metaphor and depends on the establishment of a particular correspondence between two different systems: the natural system based on empirically established causal relationships and a formal system based on inferential rules. A good model should not only map elements to elements but also processes to processes – that is, it should have congruence in its entailment structures.

However, as the American father of mathematical biophysics, Nicolas Rashevsky, pointed out in 1954, there is “no record of a successful mathematics theory that would treat the integrated activities of the organism as a whole”. In the pursuit of such a theory, relational biology employs category theory, graph theory, network theory, automata theory, computer models and formal systems. For example, the discipline often starts with an interesting mathematics structure and seeks to find natural systems that realise this structure.

In addition, the mathematical tools of evolutionary biology include population genetics, statistics and game theory.

Mathematics has clear applications in the field of biology. Biology also may help to advance mathematical studies, particularly through greater understanding of the self-fabricating property of living organisms, which persist despite the limited lifetime of their components. In one year, nearly all the atoms in our bodies are replaced, yet life persists. As a mathematics object, the cell may be seen as a perfect realisation of a formal system that can write its own production rules.

However, at present, mathematicians and bioscientists cannot really communicate properly with each other because they know too little of each other's fields. But as American mathematician Avner Friedman said: “Mathematics is the future frontier of biology and biology is the future frontier of mathematics”.

## 5.1 Discussion

Mathematics and biology are both such big, old disciplines that it is hard to train graduates in both, although the mathematics that biology students learn in their first year may not be sufficient. The mathematics that biologists need should be clearly defined and this should then be communicated clearly within the higher education system. In this regard, it is worth bearing in mind that not only can mathematics provide solutions for biology; biology can also raise fundamental questions that mathematics could usefully seek to answer. Much of the cutting-edge in research in biology is driven by mathematics.

Careful consideration should be given to the level at which the required mathematics should be taught and whether curricula should be adapted to train bio-mathematicians. A rule of thumb could be that the required mathematics should be taught when needed, which may often be at postgraduate level. Alternatively, a joint major in mathematics and biology at undergraduate level could be an option.

The usefulness of the current departmental silos at universities should be considered in terms of what mathematics is taught in which departments. This question is perhaps not so much an instrumental one, but rather one of creating greater interdisciplinary familiarity with certain mathematical approaches and methodologies. It is important to create a pedagogic culture that supports students who are scared of mathematics.

In relation to the content of courses, although the performance of certain core mathematical activities by students is crucial, it is worth considering whether too much attention is being placed on certain methodologies – for example, the relatively large amount of calculus taught in the first year at university – when broader theoretical approaches (such as modelling linked to mathematics) also require attention. The mathematical preparedness of mathematicians should be challenged.

Science, technology, engineering and mathematics (STEM) degrees and careers based on these qualifications are popular. Parents almost unquestioningly support mathematics as a subject for study. But the higher education sector may be failing to leverage such social preferences fully and may be disappointing many of those who sign up for mathematics-based degrees.

Academics teach within established curricula, which confer security. But perhaps mathematicians should be encouraged to challenge and step outside these parameters, becoming more 'vulnerable'.

The multifarious roles proposed for mathematics departments – the discipline has many spouses – raises an important capacity issue: mathematicians have to serve their own interests because they are building their own discipline – which in the field of pure mathematics may have applications that have yet to become evident. For example, the mathematics that enabled computers to function was developed long before the computers themselves were mass-produced. At the same time, mathematics departments have to service other departments and generate applicable theorems. The issue gives rise to the question of where – in which departments – the study of mathematics and its branches should actually take place.

Common ground should be sought among university departments in relation to mathematics teaching. Interdisciplinary curricula, workshops and joint activities can support this. Mathematics is a cultural asset – not just about what will work in classrooms at a university – and this should be widely addressed in science communication policies and programmes. The connections between mathematics and a wide range of socio-economic activities could be addressed in schools.

Mathematical analysis of data on the performance of schools and universities, including pass rates, could help to outline the challenges faced by the teaching of mathematics and inform policy in this area.

## 6 PLENARY 2: ECONOMIC SCIENCES

Tools developed by physics continue to shape the underpinnings of economics as a study. Both aspects of the field – microeconomics (which considers individual businesses and price theory) and macroeconomics (which considers national and global phenomena) – seek to model systemic interactions.<sup>3</sup> The approach adopted by microeconomics is axiomatic beginning with assumptions about predictable, localised behaviour and then deducing the properties of the economic system. It employs the tools of 19<sup>th</sup> century physics, equilibrium theory using calculus and some topology. Macroeconomics seeks to formulate equations for aggregate behaviour. A key concern of the discipline is how to model the rational expectations of forward-looking agents. The field uses partial differential equations and dynamic stochastic general equilibrium models.

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<sup>3</sup> Martin Wittenberg, The Role of Mathematics in Teaching and Research in Economics and Allied Disciplines, presentation at ASSAf Mathematical Sciences Workshop, Finding Synergies in the Mathematical Sciences, Cape Town, 15 - 16 September 2016.

An important concern for economics is how to model decision-making in an uncertain environment prey to uncertainty and shocks. The discipline employs a theory developed by mathematician, John von Neumann, and economist, Oskar Morgenstern, to determine how a decision-maker seeking to maximise value will behave when faced with the outcomes of different choices. This may be married to price theory to ascertain the prices of derivatives, etc., in financial markets.

The game theory developed by Von Neumann and Morgenstern in the context of the Cold War and used for modelling strategic interactions has also become a key tool in thinking about negotiation, the roles of institutions in terms of cooperation and norms, and evolution.

Implicit in all the models developed for economics is an assumption about how people ought to behave, which can be assessed by measuring how people choose, cooperate and cheat under a range of conditions and with different incentives. Psychologist, Daniel Kahneman, tested these predictions in laboratory settings and found that humans fundamentally misunderstand probability and that 'rational agent' models do not always apply. People are generally more altruistic than the model would suggest, apart from economics students who behave increasingly like rational agents the more they learn the economic view of the world. In this regard, the ways in which individuals think about the system interacts with how the system works.

The science of economics also employs econometrics, which is a branch of statistics that can provide useful data on how and why people respond the way they do in certain economic situations. A key issue in this field is the feedback loop: the people being measured often have an interest in affecting the outcome of the research.

Big data sets collected using internet search engine and social media research tools also present economists with important challenges, which can relate to privacy issues, reproducibility, measurement error and verification, as well as computational issues.

The recent financial crisis has challenged some of the tenets of generally accepted theory. For example, models often tend to assume markets operate more effectively than they do and need to take greater account of the impact of monopolies, interference such as political manipulation, and herd behaviour. Many pricing models are based on normal distribution. But the global market crash of 2008, for example, means that the factors underlying extraordinary events, including what people believe about models, need to be taken more seriously.



Despite South Africa's specific socio-economic environment and approach to development, little theoretical work is being done in the field of economics. Most is applied and not much of this is methodologically innovative. However, South African conditions could enable theorists to identify the impact of certain institutions on markets more clearly than elsewhere.

Furthermore, specific challenges are posed by implementation of the United Nations' sustainable development goals (SDGs), which set specific numerical targets for distributional objectives such as the elimination of poverty and provision of health, water and sanitation services. Achievement of the goals should be assessed not only at national, but also at local levels. Effective measurement requires the production of more statistics at a finer scale, which may be achieved by aligning a range of current data sets – large and small and of varying quality – and developing and implementing automated models for producing and analysing these.

Meanwhile, mathematics is often abused by researchers to reach tendentious conclusions. Many general equilibrium models make too many implicit assumptions and employ too many equations without divulging their content. In this regard, for example, the National Treasury has concluded from its minimum wage model that adoption of a minimum wage would result in too many jobs being cut. The University of the Witwatersrand has reached the opposite conclusion from its minimum wage model.

Mathematics at university is widely regarded as a ticket out of poverty in South Africa, where earnings inequality has risen since 1993. At the University of Cape Town (UCT), the entrance requirements for economics are stricter than for some sciences and may exceed the actual mathematical needs of first-year students. However, although many of the intake may be able to perform mathematical 'tricks' – solving quadratic equations, etc. – they do not necessarily have a feel for magnitudes and broader relations or an understanding of how to translate real economic problems into the formal language of models and back again.

Many of the mathematical models – for example, relating to supply and demand curves and how these may shift – are presented graphically because students lack the algebra and calculus skills to solve the simultaneous equations that underpin them. Students have difficulties understanding probability. Students also often seem unable to think analogously – for example, by converting verbal descriptions into appropriate mathematical models.

In addition, students confuse and substitute tools and models for describing reality, which are based on hypothetical reasoning, with reality itself – often seeking to apply them with a missionary zeal and disregard for actual conditions. Furthermore, the models that undergraduates are taught may be simplifications – such as demand and supply – and the complete, more complicated models are often only taught at postgraduate level.

Schools often fail to prepare students properly for economics courses at university. Many of the economics ideas and approaches taught at school have to be unlearned by students at university.

## 6.1 Discussion

Mathematics ability may be considered a proxy for abstract thought. However, mathematics courses alone may not be able to inculcate such understanding, or at least may need to be adapted to do so. In this regard, the relationship between performance in mathematics and natural language ability should perhaps be explored and more clearly defined. Notwithstanding the importance of mathematical thinking over mathematical performance, it must be stressed that mathematical manipulation is absolutely necessary to the discipline – students need to be able to use the symbols properly.

The mathematics grades achieved at school do not necessarily immediately translate into levels of adeptness. Schools should seek to focus on developing the disposition of pupils as mathematical actors through living mathematics in order to inculcate greater problem-solving and critical analysis skills. Students have widely varying views of themselves as mathematics practitioners when they enter university. The challenge for universities is to build critically but constructively on students' sense of self-worth.

Students should graduate with a nuanced understanding of the economic models to which they have been exposed, rather than a 'missionary' view of the world. Problem-solving is fundamental to effective modelling and graduates should be able to reflect on the contexts of the problems with which they are dealing. The reasons why the concept of probability seems so hard to teach should be researched. The use of more complex models may help in communicating the idea more effectively.

Large-scale changes to curricula may not provide solutions to the issue of integrating mathematics more effectively in related disciplines. In order to improve the outcomes, it could rather be considered preferable

to modify the exams themselves – and thus the syllabus. Pedagogic practices also can be changed to improve outcomes – although such change can be difficult to achieve.

University departments have recently been cramming more and more content into their science and mathematics degrees. By contrast, some arts degrees have focused rather on raising standards of writing and thought and have trimmed their content accordingly. Science and mathematics curricula may learn from this and adopt a ‘less is more’ approach without dumbing down.

There is a lack of articulation between the research conducted by economics and mathematics departments at universities and the models that are required by the financial sector in the broader society. Closer relations and communication may usefully be established between mathematics and economics departments and the financial sector, including banks and insurance companies, to address these research concerns, including in actuarial science, and to raise educational standards. Furthermore, economics and mathematics departments should collaborate more closely within universities.

It may be worth describing what kinds of mathematics are most needed to address South Africa's developmental problems. ASSAf should identify the kinds of philosophy and materials that it plans to develop as a champion of change in the integration of mathematical studies, as well as the key stakeholders that it should partner in this process.

## 7 PLENARY 3: MATHEMATICS AND BIG DATA

South Africa should assign a body – perhaps the National Research Foundation (NRF) – to conduct foresight studies to identify areas of economically important innovation and new trends in mathematics.<sup>4</sup> Notwithstanding the absence of such a national study since the 1990s, the intertwined areas of computation and big data have emerged as major drivers of mathematical research. Expertise in simulation and large-scale data analysis has increasingly become required in major research efforts.

Big data may be defined as massive data sets that are so voluminous and/or move so fast and/or vary in quality and structure to such an extent that they cannot be stored and processed using traditional

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<sup>4</sup> Loyiso Nongxa, Mathematical and Statistical Challenges of (Big) Data Sciences, presentation at ASSAf Mathematical Sciences Workshop, Finding Synergies in the Mathematical Sciences, Cape Town, 15 - 16 September 2016.

database methods and software. Special analytical tools and processes – software programmes and their underlying algorithms – continue to be developed to examine and extract useful and valuable information and knowledge from such data.

Mathematics plays a crucial role in many aspects of big data by, for example, enabling the quest for influential nodes in huge networks; segmenting graphs into meaningful communities; modelling uncertainties in trends; linking databases with different levels of granularity; helping to produce unbiased sampling; and connecting with censorship infrastructure to protect privacy.

South Africa should found its big data analysis projects on the computational, mathematical and statistical sciences, by contrast with the United States where discussions about big data tend to focus on business analytics. The field is evolving rapidly – today's big data may be considered small data in ten years' time.

Analysis of big data poses a range of challenges. These include the data's high dimensionality (many parameters); heterogeneity (structured or unstructured, numeric, pictorial, text-based, etc.); scale (large); timeliness (quick, dirty solutions are often preferable to slower, more elegant ones); and confidentiality (legal and ethical issues around privacy).

The areas in which the field is developing rapidly include: scalability (the extension of existing theories and algorithmic techniques to new, larger scales); new theories; and novel applications. The mathematical sciences will influence developments in big data analysis and vice versa. For example, randomised numerical linear algebra can be used to sample columns of a large matrix judiciously to extract a useful, smaller one. Key research areas include: topological data analysis; matrix and tensor decompositions; dynamical networks (graph theory in which nodes come and go, and edges may crash and recover); random graphs (at the intersection of probability and graph theories); statistical learning (the use of automated and semi-automated techniques to predict user-relevant quantities); computational statistics; optimisation; Bayesian statistics; and applied harmonic analysis.

Big data studies may have particular eResearch applications in South Africa, including in astro-informatics (a new discipline being developed as part of the Square Kilometre Array [SKA] project); bioinformatics; particle physics research (conducted by South Africans at the facilities of the European Organisation for Nuclear Research [CERN]); business analytics (how to make money by mining data); finance (mainly in the banking sector and for applications analysing stock price variations,



etc.); telecommunications (spreadsheets for monitoring data usage, etc.); insurance (actuarial sciences); marketing; and retail.

Big data studies can also help the government by strengthening the capacities of Statistics South Africa and the South African Revenue Service (in a new tax analytics unit investigating money laundering, etc.). In the health care sector, private health providers are improving efficiencies (and making money) through modelling based on big data. In the social sciences, big data from social media outlets, for example, can be used to study the relationships between individuals' environments and their thoughts, opinions and actions.

Against the background of fragmented mathematical sciences, big data studies offer an opportunity for mathematical scientists to communicate with each other about synergies among their disciplines, as well as opportunities for collaboration among researchers across departments and with users of mathematics outside universities (in business and government). The new field also offers an opportunity to produce graduates who eschew early specialisation in favour of a greater breadth of knowledge.

However, the new discipline may also spawn territorial claims that result in narrowly focused programmes, as well as a proliferation of low-quality qualifications that could undermine the confidence of external stakeholders. By contrast, the discipline could also foster profitable, data-driven innovation (for example, the algorithms that companies such as Google buy to improve and expand their business models).

Mathematical sciences should position themselves at the core of developments around big data. This could help to rejuvenate these disciplines in South Africa.

## 7.1 Discussion

A central concern is how to adapt, and not just re-label, courses to include data analysis. The curriculum should include mathematics and computer sciences. It may be easier to do this at postgraduate level. Collaboration among departments is important to achieve this but the most important concern is to ensure that the fundamentals are taught properly.

The courses should teach some algorithmic and probabilistic thinking, although computers have taken over many algorithmic functions. In relation to probabilistic thinking, flexible entry into statistical studies at

postgraduate level may provide a solution. Technical questions over which data to store and ignore may also need to be addressed.

The question of how data science fits into university programmes may often be shaped by present priorities and structures. For example, in the Western Cape, data science is often equated with astrophysics because of the SKA project. At the University of Pretoria, one group of computer science students resides in the engineering faculty and a second group in the statistics department. At Stellenbosch University much of the big data activity is in the life sciences.

It is important to ensure that the mathematical sciences remain involved in big data studies, although mathematics should not subordinate itself to the new field. Higher education institutions should seek to integrate the topic in a way that complements existing studies. The field of enquiry may present a greater opportunity to some institutions than others. In this regard, although collaboration within the higher education sector and with appropriate external actors is important, the crucial issue is to ensure that the fundamentals for big data studies are taught properly. This may also entail training schoolteachers how to equip their students with the appropriate skills and approaches.

At some universities it is not possible to pursue majors in both mathematics and statistics, both of which are essential for big data. A solution to such curricular challenges may be to add short supplemental courses to the relevant degrees. Alternatively, undergraduate mathematics courses could include some statistics. In general, mathematics students should be encouraged to acquire new knowledge and be fleet of foot.

The issue of funding for big data studies also needs to be addressed. At present, full funding is only granted if a degree consists of more than just coursework and has a research component. In this regard, it is noteworthy that the African Institute for Mathematical Sciences (AIMS) South Africa has established a chair in big data to consider issues of signal analysis.

The issue of big data, like other mathematical specialisations, may best be addressed at postgraduate level. In this regard, undergraduate mathematics should avoid early specialisation and offer a greater smorgasbord of courses.

Big data courses must include modules on data mining which should foster implicit skills, such as visualisation and adopting a critical approach, in order to develop useful statistical models.

The discipline may also borrow tools used in economics such as graph theory to consider problems of positioning – whether a particular position has meaning. Stochastic calculus employed to analyse sociological structures may also be used within big data.

The study of big data raises sociological and ethical concerns on its role in the wider world, particularly in terms of South Africa's development. However, big data as a field of study should not only be application-driven and should support the study of mathematics for itself.

It is important that mathematicians and scientists come together to discuss the impact of big data as a research methodology and area for research. Within the field of higher education itself, the use of big data may enable more effective planning – for example, in the production of more appropriately tailored courses.

## 8 PLENARY 4: PHYSICAL SCIENCES

The relationship between mathematics and the physical sciences, including astronomy and technology, is longstanding and traces its roots back to antiquity. Examples may be found among the ancient Chinese, Greeks and Hindus, and the Incas, to mention a few.<sup>5</sup> From around the 17<sup>th</sup> century there were dramatic developments in these relationships that were fuelled by questions arising in astronomy, mechanics, heat conduction (for example, French mathematician Joseph Fourier's work) and electromagnetism (through the major figure of James Clerk Maxwell). It is important also to mention the early links between mathematics and what was to become computer science, for example in the investigations of Ada Lovelace, who worked on Charles Babbage's mechanical computer. The last century has witnessed explosive growth in mathematics and its relationship particularly to physics. So, for example, astrophysicist Subramanyan Chandrasekhar won recognition, including the Nobel Prize in Physics in 1983, for his mathematical treatment of stellar evolution.

Mathematics continues to have a two-way relationship with the physical sciences. Thus, it is not simply a tool or servant in this relationship. On the contrary, problems in the physical sciences provide fertile ground for the development of new mathematical techniques and theories. In this regard, the challenge today is to acknowledge the reciprocal nature

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<sup>5</sup> Daya Reddy, Synergies between Mathematics and the Physical Sciences, Astronomy and Engineering, presentation at ASSAf Mathematical Sciences Workshop, Finding Synergies in the Mathematical Sciences, Cape Town, 15 - 16 September 2016.

of this relationship as central to determining the nature of teaching and research in mathematics.

The interwoven nature of mathematics with the physical sciences may be appreciated through a concrete example, from materials science and mechanics, which is concerned with the behaviour of metals subjected to a variety of loading processes. The first step in developing a theory with predictive capabilities is that of the mathematical model, which in this case would take the form of a system of partial differential equations and inequalities. The model is then analysed for its well-posedness. Given its complexity, computational methods are then essential for obtaining approximate solutions. These methods would require the development, analysis and implementation of appropriate algorithms.

The results of such a study may then throw up features that were not apparent in prior experimental work. And, as has been the case for this area of research, the study of these features may highlight the need for completely new mathematics in order to analyse the special features. Thus there has been major progress in the development of new function spaces that would be the appropriate setting in which to study phenomena in metals, such as slip lines, which are discontinuities in the deformation.

The cycle thus returns to the starting point, in which the modelling process is revised, this time with the availability of an enriched mathematical theory.

This extremely applied example illustrates also why the dichotomy between pure and applied mathematics is obsolete and retrogressive, whether with regard to teaching or research. The spectrum of mathematical activity is a broad one and it is important that barriers not be erected to hinder fluid movement back and forth along this spectrum.

The American mathematician, Robert Zimmer, likens mathematics to a fabric – a woven artefact that derives its strength from its interconnectivity, but which may be weakened by a tear or gap. Alternatively, mathematics may be seen as a grand edifice, such as a cathedral, the construction of which is the result of contributions, many of them small, but all important and integral to the overall creation.

In promoting actively the relationships among mathematics and a multiplicity of disciplines, it is important to develop an understanding not only of technical content but also of the various cultures and methodologies inherent to areas such as chemistry, biology, and engineering. Such an approach would pave the way to identifying



interesting and important problems at the interfaces between these disciplines and mathematics.

The emergence of data science, a product of the digital revolution, poses a new all-encompassing challenge. Global information storage capacity has grown rapidly since 2000, resulting in what is referred to as big data. This environment provides unprecedented opportunities to simulate complex systems dynamics, for example, by discerning through analysis of the data hitherto unseen patterns and unsuspected relationships. The data revolution offers abundant new opportunities for mathematics and it is important to prepare students so that they are able to respond successfully to these opportunities, whether in research or in a range of careers.

Curricula in mathematics should be informed by its interactions with a range of areas. Such considerations may lead to new courses, majors, and serious partnerships with other disciplines. Computation is central to modern mathematics and its relationship to the physical and other sciences: as an aid to discovery, a way of seeking new insights, and in order to solve problems that are otherwise intractable. It should be an integral part of every mathematics curriculum.

The historical distinction between statistics and mathematics is unproductive and should be challenged. There is so much that is hardly classifiable as strictly mathematics or statistics, for example, in the important area of uncertainty quantification. Likewise, mathematics is the poorer for an absence of probability in its curricula.

Continuous reflection is required on what constitutes a well-rounded mathematics graduate, as well as the most appropriate structure of mathematics curricula for physics and engineering students. Courses for students in the physical sciences should offer not only the tools and the expertise, but the ability to adapt and build, and to discern new ideas and formulate problems. The cultural values of mathematics, including its beauty and notions of rigour, are as important components as ever in any curriculum for the subject.

## 8.1 Discussion

Courses need to do more than just teach the mathematical 'recipes', many of which are never used again – they should teach more ideas and inculcate greater problem-solving. This may be done by, for example, setting students key problems at the beginning of their courses – the challenge being to solve them later as they are equipped with the appropriate tools.

University teaching staff should pay greater attention to international developments in interdisciplinary mathematics and should also update their courses to include more recent developments and discoveries in their fields, as well as skills in newer areas such as quantum information and computing. In this regard, staff may need to acquire greater familiarity with basic computing packages such as Python.

Greater communication on interdisciplinary concerns should be fostered within universities (for example, among autonomous units within departments), among universities and with relevant non-governmental and governmental bodies. Strong territoriality in university departments must be challenged.

Honours degrees should be more flexible in their content. More degrees across departments should be developed for students looking at a range of subject areas. At UCT, dual degrees are being considered. More well-rounded staff conducting research and teaching across disciplines would support this process.

The artificial divide between mathematics and applied mathematics is not beneficial for students. In this regard and to overcome compartmentalisation, the structure of science faculties as a whole should be reassessed.

The problem of compartmentalisation extends to the management of university education itself. At present, many university managers and government officials are not prepared to listen to new ideas about how mathematics and sciences should be taught at universities. One way of attracting their attention may be to define the current interdisciplinary problem as part of the process of decolonising higher education and so enabling our students to be serviced properly.

The issue of universities' resistance to change on curricula issues should be addressed and whether it, perhaps, constitutes a reluctance to move away from teaching the same content that academic staff themselves were taught as university students. Resistance to change may also stem from structural factors – universities tend to take the long view, looking back a long way as well as forward – as well as the specific colonial formation of these institutions in South Africa with their inherited power structures.

In terms of universities' role in society at large, mathematics and science departments should show that they can respond proactively to shaping the agenda for change in higher education and take a broader look at transformation – moving it beyond discussions about classroom and

assessment practices to address South Africa's broader social reality and developmental needs.

The student protests may have had the effect of pushing some academics back into the laager, obliging them to defend their bastions of privilege against some of the attacks on fundamental liberal educational values. In this regard, the academic community needs to reconnect to a progressive vision, which has been captured by other forces and an often overwhelmingly instrumental discourse.

Some leadership for change in mathematics and science studies should come from the faculties and the broader science community. The process should be a proactive and collaborative one, engaging students but not defined by their discourse. The professional and advisory bodies have a key role to play both in helping to convene the process and in shaping the broader consciousness to support it.

## 9 PLENARY 5: MATHEMATICS EDUCATION

Mathematics plays a substantial role in teaching and research within both the physical and social sciences. Developments in research in engineering, biology, economics and mathematics have implications for an undergraduate curriculum in the mathematical sciences. However, two further critical questions need to be considered.<sup>6</sup> Where in a discussion of the future of the mathematical sciences in a rapidly changing and challenging and exciting world, should the career of the future school mathematics teacher be located? What does this location mean for a mathematical sciences curriculum or education in the university? In the 2025 review of the mathematical sciences in the United States, the importance of teachers for K-12 was addressed in a small section, with discussion about the top mathematics school-leavers entering/competing for teaching careers in high performing countries; and so too a silence on the mathematical preparation of teachers as part of the mathematical sciences. This is important to think about – specifically in terms of such reviews in high performing countries. The question is: Would mathematics for teaching be an area for consideration alongside mathematics for biology/finance/social science, as discussed in earlier sessions. What is different? What is taken for granted?

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<sup>6</sup> Jill Adler, Role that Mathematics Plays in Education, presentation at ASSAf Mathematical Sciences Workshop, Finding Synergies in the Mathematical Sciences, Cape Town, 15 - 16 September 2016.

There is increasing agreement that there is a specificity to the mathematical knowledge that is required and used in teaching work which many learn on the job. But there is a lack of agreement about definitions, the language used to describe this specificity and what are its basic concepts. There is also less consensus on the boundaries between this knowledge, mathematics itself, and mathematics pedagogy and consequently on the best practices that can be adopted in the education of primary and secondary school mathematics teachers. Large-scale research studies have found a moderate association between teachers with appropriate knowledge of mathematics and pedagogical training and improved teaching and learning. Also, if a teacher's advanced mathematical ability exceeds a certain threshold, it produces negligible improvements in learner outputs. Furthermore, in terms of the usefulness of what is being taught, it has been found that calculus coursework enhances learner achievement in algebra but not necessarily geometry.

Teaching mathematics thus entails specialised ways of knowing mathematics. This can mean different types and thresholds of mathematical knowledge for teachers at the various levels from pre-primary to tertiary. The required knowledge needs to be ascertained and defined. Decisions should also be made about when and where such knowledge should be taught.

The present routes into teaching mathematics are: a Bachelor's degree in (or at least with some) mathematics taught in mathematics departments in universities, followed by a Postgraduate Certificate in Education (PGCE) taught in education departments; or a Bachelor of Education degree, with mathematics and education courses taught predominantly in education departments. Both models have their constraints. The former path, in which all specialised mathematical knowledge for teaching is condensed into one year in the PGCE, may provide insufficient knowledge of and for teaching geometry, statistics, probability and financial mathematics, which are part of the school, but not necessarily the undergraduate, mathematics curriculum. As noted, research has shown that the calculus taught at university does not support the teaching of geometry at school as it does algebra. In addition, persistent problems such as, for example, a clear understanding of what does and doesn't constitute a mathematical object may not always be addressed along this path.

The mathematics taught as part of the BEd has different limitations. Its intake of students is also different with many entering having passed mathematics in Grade 12, but with mathematical knowledge that is often ritualised, meaning they are able to execute procedures and with some skill but not grasp mathematical principles underlying these



processes. There is thus a need to revisit school mathematics and to do so from an advanced perspective, deepening what prospective teachers know and understand about school mathematics that is critical for teaching, including specialised knowledge of all the domains that comprise the school curriculum. For example, if you want teachers to be able to think probabilistically, then you need to include in the mathematics programme a course offering the key ideas, concepts and processes they need to know and be able to do. And this needs to be done while introducing and developing higher level mathematics and courses in relevant pedagogy. This is a far broader curriculum, just mathematically speaking, and thus cannot also provide for depth and extension across domains as the current, dedicated three-year degree in tertiary mathematics.

To address some of the challenges faced in the classroom, the Wits Mathematics Connect Secondary (WMCS) project at the University of the Witwatersrand developed and implemented a professional development programme to improve the teaching and learning of mathematics in ten secondary schools in one province. The programme sought to enable in-service teachers in disadvantaged schools to improve their mathematical knowledge for teaching and to construct and enact lessons that achieved their stated mathematical goals. The programme was particularly concerned with addressing a significant blockage in the education pipeline that takes place between grades 9 and 10, although the pupil pipeline can be a problem from grade 1. Those teaching grades 8 and 9 are often under or unprepared, leaving teachers in grades 10 and 11 with an almost insurmountable task as they seek to play catch-up – re-teaching with too few resources in poor schools. As a result, many pupils fail at grade 10.

The WMCS professional development programme includes a 16-day mathematics for teaching course focused on content relevant to the Grade 9 - 10 transition. Teachers attended eight two-day sessions over the course of a year. These focused on functions, algebra, geometry and trigonometry and included a lot of independent coursework in between. The topics were chosen according to their relative importance within the curriculum, as well as for their potential to deliver leveraged learning gains.

The course revisited mathematics known to the participants and strengthened and extended this existing knowledge. For example, extreme cases were explored and aspects of mathematics that had been taken for granted were problematised. Connections were made across the curriculum. New mathematics was taught beyond the curriculum and the grade-level taught. The teachers worked on their

knowledge of key concepts and were trained to be fluent in the essential procedures, which were practised. In pedagogical terms, the issue of adopting a language that traversed colloquial and mathematical terms to communicate effectively was addressed. An understanding of the role and importance of mathematical proofs was developed. The teachers learned that the how, what and the why of mathematical models have to be understood and taught.

The impact of the professional development course was measured by assessing the learning gains among a cohort of 609 pupils in five schools over an academic year. The learners taught by the teachers who had taken the course significantly outperformed the learners in the same schools taught by teachers who had not taken the course. This was a pilot study and the results were indicative only. Nevertheless, the implications are that enhancing teachers' mathematics for teaching can lead to improvements in learning. For the WMCS project, which is now in its second phase, the goal is to extend the reach of the programme and research sustainable ways of doing this.

The study has important implications for the shape that may be taken by a well-rounded undergraduate curriculum that includes mathematics teacher education. Broadening curricula inevitably has consequences for working within and across disciplines. At the same time as there would need to be a weakening of the boundary between mathematics and the teaching and learning of mathematics, so there would need to be specialisation in mathematics. This tension between a broader curriculum and depth in disciplinary specialisation has to be managed. In addition, as a specialism in its own right, mathematics for teaching should also seek to nurture the identities and expertise of 'educators', who may foster the next generation of mathematics teachers.

## 9.1 Discussion

Mathematics departments need to address the issue of how to encourage students into mathematics teaching as a career and equipping them with the required skills. This would also encourage pedagogy in these departments that constructively addresses the shortfalls in conceptual understanding and knowledge among many new students and may promote an accompanying cultural shift among faculty staff.

Since university-teaching tends to be personality-driven, a move within mathematics departments away from the often intimidating and, perhaps, isolationist world views of pure mathematicians, to a broader understanding of society's imperatives and demands may be required to foster graduates with a broader understanding of their discipline.

The extent and kind of integration between education and mathematics/science departments needs to be reviewed carefully. Universities may also be able to shape the mathematics/science input into teaching diplomas. The standards of BEd courses, which are lower than those for other courses, should be raised.

The number of engineering students dropping out of the faculty in the first two years of study is great. Perhaps some of these students could be diverted into teaching, leveraging the greater availability of bursaries for BEd students.

One four-year degree model that has been successfully adopted and caters to hundreds of students in South Africa mixes science and mathematics courses with an education component: students do the first two years of a BSc and then a third year of BEd which includes a review of secondary school mathematics, before returning to the BSc for the final year. Most of these degrees are offered with mathematics alone or with mathematics plus computing or physics.

The status of mathematics teachers in South Africa needs to be raised to encourage more university students to enter the field. Previously, five per cent of the cohort at universities would become teachers and train the next generation, but this is no longer the case and teacher shortages are mounting. The problem may be exacerbated by university mathematics departments relying on their postgraduate students for teaching at university. Greater incentives should be offered to mathematics graduates to go into teaching in schools. For example, the non-profit organisation, TEACH South Africa, takes graduates and places them in schools where they are mentored and take a PGCE course. The establishment of a system of live-time virtual classes could also enable more teachers to learn from appropriate role models.

The benefits of the many intervention programmes for in-service teachers that are continually conducted in South Africa (with a lot of investment from the state, the private sector and aid organisations) need to be properly evaluated and collated. The programmes that are found to succeed should be rolled out, although there are many impediments to scaling up such projects, including from the relevant government departments.

Short-term interventions are insufficient to address the challenge of teaching mathematics in schools effectively. Teachers need time to learn mathematics again, if they are to teach it properly. In this regard, mathematics education research tends to focus on content or pedagogy. But it has been repeatedly shown that it is when you

integrate the two that projects have genuine impact. The question of the depth of mathematical understanding and training required by a teacher is important.

Lessons may be learned from other African models for teaching in schools, many of which employ traditional pedagogy. In Zimbabwe, teachers are imbued with a sense of what mathematics is, as well as a sense of how it should be performed (not a mere set of conceptual skills) and a professional approach to teaching. Many Zimbabwean teachers participate in mentoring programmes. However, some of the pedagogical approaches adopted in African classrooms outside South Africa, in which there are often fewer girls and less student interaction, may not be transferable.

## 10 CONCLUSION

Although mathematics is fundamental to the understanding, creation and use of much knowledge within all the mathematical sciences, as well as for teachers of mathematics, the formal relationship between the discipline of mathematics and other disciplines and broader social functions may be characterised as tenuous. The exclusivity and remote nature of the practice of mathematics (and some sciences) can discourage many students and would-be practitioners. The idea that those applying for economics and, to an extent, medical degrees should demonstrate a certain facility in mathematics beyond that actually required to complete the course purely in order to control the number of entrants, is problematic. In addition, low salaries for schoolteachers (their wages used to be comparable with those of university lecturers) may dissuade many graduates from entering this particular profession.

The study and use of mathematics raise important issues of disposition, identity and culture – not just knowledge – among students. For example, engineering students tend to say “I am an engineer” from their first year at university. But mathematics students think they have to make some important discovery before calling themselves “mathematicians”. In this regard, the purpose of undergraduate studies could be assessed more holistically.

Universities' biggest social impact is in their teaching. Students, particularly in the first year, should be encouraged to define their academic and other goals and fully engaged in their studies by their teachers in pursuit of these goals. The pedagogy should excite and inspire.

Broad cultural prejudices that promote the view that mathematics ability is innate should be overcome both at universities and within



the wider society. Mathematics should be taught in a welcoming way that encourages everyone to learn – the subject should be framed as democratically available and not just the preserve of a predisposed elite. An open culture of mathematics needs to be built starting from the pre-school level.

Role models – people with a genuine love and enthusiasm for their own subjects – may be able to break down the barriers of prejudice. TED-type talks may also stimulate pupils and students. Top scholars can exemplify an inspirational way of teaching through the reverence for knowledge that they often display.

More broadly, including in the mathematical sciences, universities need to address poor throughputs – only about half of students are completing their degrees at present. In this regard, the kind of mathematics teaching on offer may be regarded as insufficient to the task. The question of student identities and how these can be undermined by assessment and pedagogical processes needs to be considered. However, discussions about decolonisation in the higher education sector need to be nuanced so that a full view may be achieved of what must be retained and what must be changed.

External economic factors, including employability and bursaries that may be available for some specialisms (such as actuarial sciences) and not others, also impact the graduate and postgraduate pipelines.

Mathematics is crucial to many new ways of making knowledge in biology, economics, physics and engineering, etc. The need for specialist mathematics may be addressed in a range of ways from picking up new skills on an *ad hoc* basis at the postgraduate level to implementing radical changes to curricula. The kinds of mathematics that are taught should not merely be tailored to the demands of business and industry, but should also help to sustain the educational enterprise in terms of producing future researchers and teachers.

Departmental silos within universities enable specialist academic disciplines to wield power and provide some bureaucratic protection. However, they also hamper progress towards integration of mathematical studies across disciplines. The research topics chosen and supported in departmental silos are often driven by turf politics, self-promotion, the availability of research grants and the need for National Research Foundation approval. This can militate against the production of valuable and relevant mathematical work in South Africa.

Mathematics departments alone should not bear the brunt of silo-isation in universities – other departments must also contribute in terms of teach-

ing mathematics. In addition, the conceptualisation of the individual mathematical sciences as discreet disciplines can impede the development and implementation of interdisciplinary projects. Mathematics should not be regarded as a discipline purely at the service of the other disciplines – it is a two-way process.

In this regard, curricula may need to be restructured. Greater flexibility within curricula and joint interdisciplinary projects may also offer ways forward. Cross-disciplinary inputs should also be reassessed – for example, engineering students are taught a lot of calculus but little discrete mathematics (such as graph theory, etc.), which are quite easy to teach. Departments need to review which mathematical tools from the huge variety available they should be teaching.

Moving from mathematical models to real-life situations is the biggest challenge faced by many students. Students should be equipped with the disposition to seek out new knowledge and apply their mathematics skills accordingly. Their studies should help them to feel more at home in all the worlds – the academic, the natural, the socio-economic, etc.

Issues with practical application of mathematics skills can be traced back to the kinds of mathematics teaching experienced by many students before university, at school, where many pupils are only shown mathematical models rather than being taught the mathematics behind them. The systemic design of the schools system, in which at least seven subjects are taken at matriculant level, may impede higher achievement in mathematics and the sciences.

In addition, the question of learning the right kinds of mathematics to teach is crucial for graduates planning to become educators. Breadth rather than depth (less is more) may be a useful guiding principle, although research mathematicians can also contribute valuable insights to mathematical understanding that may enhance pedagogy in schools.

## 11 WAY FORWARD

In the quest to foster greater synergy among the mathematical sciences within universities, the issue may be less one of possible departmental mergers and more one of creating a culture in which mathematicians (of whatever specialisation) talk to each other freely. If this mindset is fostered and such communication happens, this could filter through into the curriculum and perhaps foster more and greater cross-disciplinary inputs. One way of structuring the debate would be to consider the nature of the ideal end-product that is sought – a well-rounded mathematics/science graduate – and then work backwards.

It can be difficult to change complex systems – such as mathematical and science teaching and research at universities. Broad changes such as large-scale curriculum reform may not have the desired effect. An alternative would be to employ many small safe-to-fail experiments and pursue further those that seem to work.

A study on such interventions in South Africa may be useful in this regard.

A culture of greater collaboration may also be fostered in mathematics and the sciences by large-scale democratic projects conducted via the internet and social media, such as the production of open-source software or the polymath project, in which all and sundry are invited to join in and contribute to the solving of often relatively intractable, complex problems. This drive may also take the form of specific country-based initiatives. For example, in France, the Academy of Sciences launched a Houses for Science project in 2012. Each of the nine regional houses seeks to help teachers in their respective area bring innovation to their science-teaching practices.

Although a more open mathematics culture cannot be called into existence by declaration, its creation and change can be supported through the commitment and leadership of key individuals, in particular among the academic community. However, working together across institutions without a declared common purpose, on a case-by-case basis, can be difficult. In this regard, drivers for broader change within higher education, such as ASSAf, have a crucial role to play.

In recognition of the broader needs for mathematics in society, any outcome study or report based on the presentations and discussions at the ASSAf workshop on Finding Synergies in the Mathematical Sciences should address bridging the gap between academia and society both in relation to the pedagogies and subject areas of the mathematical sciences. It should seek to present a comprehensive, nuanced account of the views of those working within the individual mathematical sciences and in mathematics education.

It should be endorsed by the subject societies and other relevant role players in order to carry sufficient authority. It should be noted that ASSAf studies have garnered respect in this regard. A report from the meeting may be used as a springboard for a longer consensus study.

A brief, accessible version should be produced for maximum impact with policy and decision-makers to challenge consciousness about how best to deliver mathematics.

This report may be used to engage the Department of Science and Technology, which recognises the need for investment in this area, about strengthening the basic sciences. It may also be used to increase awareness of the issues among the networks of those attending the meeting, particularly university staff who actually shape curricula. In this regard, it is crucial to address academic and student cultures at universities in order to achieve change.

The dissemination of the report and implementation of its contents may be driven by a panel consisting of members drawn from: the field of mathematics; the field of mathematics education; the mathematical sciences; the broader scientific community; industry; informal science and mathematics platforms, such as science centres, museums, etc.; and curriculum and policy areas.



## ANNEXES

### Programme

#### Finding Synergies in the Mathematical Sciences

The workshop is structured around five themes, each with a similar format. Each theme embraces a cluster of disciplines. In each theme, an invited plenary speaker will open up a discussion space that specifically explores the role that mathematics plays in the teaching and research of some or all of the disciplines listed. The 30-minute plenary talk will further provoke ideas about the notion of seeking synergies between the mathematical sciences across the academic project. After each plenary talk, there will be an opportunity for breakaway groups to brainstorm and unpack the plenary talk in terms of its objectives and provide feedback on their discussions to the collective group. Each theme is chaired by a member of the ASSAf STEM Committee who will introduce the plenary speaker, manage the report-back session of the breakaway groups to the plenary, chair the ensuing plenary discussion and keep time. A scribe will capture the essence of each of the plenary presentations and report-back sessions.

At the end of the workshop there will be a panel discussion that will seek to identify synergies and chart a way forward. The panel consists of the four spokespersons elected by each breakaway group.

#### THEMES

<b>EARTH AND BIOLOGICAL SCIENCES</b>	Geology, Geography, Environmental Science, Botany, Molecular Biology, Medical and Agricultural Sciences.
<b>ENGINEERING AND PHYSICAL SCIENCES</b>	Physics, Chemistry, Astronomy, Engineering Sciences.
<b>ECONOMICAL SCIENCES</b>	Economics, Accounting, Business Sciences
<b>MATHEMATICS</b>	Pure and Applied Mathematics, Statistics, Computer Sciences
<b>EDUCATION</b>	Mathematics Education, Science Education, Social Sciences

**DAY 1**

Thursday, 15 September

09:00 - 09:30	Registration	
<b>OPENING SESSION</b>		
09:30 - 09:40	Welcome	<b>Fritz Hahne</b> Chair: ASSAf STEM Standing Committee
09:40 - 10:00	Structure of the Workshop	<b>Marc Schäfer</b> ASSAf STEM Standing Committee
<b>THEME 1 – Chair: Marc Schäfer</b>		
10:00 - 10:30	<b>PLENARY 1</b> Earth and Biological Sciences	<b>Jannie Hofmeyr</b> University of Stellenbosch
<b>10:30 - 11:00</b>	<b>Tea</b>	
11:00 - 11:45	Breakaway Groups: Response and Discussion to Plenary 1	<b>All</b>
11:45 - 12:15	Report Back of Breakaway Groups to Plenary	<b>Breakaway Group Chairs</b>
<b>12:15 - 13:30</b>	<b>Lunch Break</b> <i>Participants are encouraged to continue interacting with each other and possibly sit with people who are not members of their Breakaway Group.</i>	
<b>THEME 2 – Chair: Zodwa Dlamini</b>		
13:30 - 14:00	<b>PLENARY 2</b> Economical Sciences	<b>Martin Wittenberg</b> University of Cape Town
14:00 - 14:45	Breakaway Groups: Response and Discussion to Plenary 2	<b>All</b>
14:45 - 15:15	Report Back of Breakaway Groups to Plenary	<b>Breakaway Group Chairs</b>
<b>THEME 3 - Chair: Fritz Hahne</b>		
15:15 - 15:45	<b>PLENARY 3</b> Mathematics	<b>Loyiso Nongxa</b> University of Witwatersrand
<b>15:45 - 16:15</b>	<b>Tea</b>	
16:15 - 16:45	Breakaway Groups: Response and Discussion to Plenary 3	<b>All</b>
16:45 - 17:15	Report Back of Breakaway Groups to Plenary	<b>Breakaway Group Chairs</b>
<b>Consolidation and Discussion – Chair: Marc Schäfer</b>		
17:15 - 18:00	Day's Consolidation	<b>All</b>
<b>End of day 1- announcements</b>		

**DAY 2**

Friday, 16 September

<b>Theme 4 – Chair: Marietjie Potgieter</b>		
08:30 - 09:00	<b>PLENARY 4</b> Physical Sciences	<b>Daya Reddy</b> University of Cape Town
09:00 - 09:30	Breakaway Groups: Response and Discussion to Plenary 4	<b>All</b>
09:30 - 10:00	Report Back of Breakaway Groups to Plenary	<b>Breakaway Group Chairs</b>
<b>Theme 5 – Chair: Kevin Govender</b>		
10:00 - 10:30	<b>PLENARY 5</b> Education	<b>Jill Adler</b> University of Witwatersrand
<b>10:30 - 11:00</b>	<b>Tea</b>	
11:00 - 11:45	Breakaway Groups: Response and Discussion to Plenary 5	<b>All</b>
11:45 - 12:15	Report Back of Breakaway Groups to Plenary	<b>Breakaway Group Chairs</b>
<b>Panel Discussion – Chair: Jenni Case</b>		
12:15 - 13:15	Panel Discussion with Collective	<b>All</b>
<b>13:15 - 14:00</b>	<b>Lunch</b> <i>Participants are encouraged to continue interacting with each other and possibly sit with people who are not members of their Breakaway Group.</i>	
<b>Consolidation and Way Forward – Chairs: Fritz Hahne and Marc Schäfer</b>		
14:00 - 15:00	Key findings and discussion points will be presented and next steps identified with a particular focus of whether an in-depth consensus study is needed.	<b>All</b>
<b>Departure and End of Workshop</b>		

# PARTICIPANTS ATTENDANCE REGISTER

## DAY 2

Thursday, 16 September 2016

#	SURNAME	NAME	INSTITUTION
1	Adler	Jill	University of the Witwatersrand
2	Barnard	Barrie	AIMSSEC
3	Case	Jenni	University of Cape Town
4	Chibisa	Admire	University of Zululand
5	Chule	Siyabonga	Stellenbosch University/African Institute for Mathematical Sciences
6	Conana	Honjiswa	University of the Western Cape
7	Diab	Roseanne	ASSAf
8	Dlamini	Zodwa	Mangosuthu University of Technology
9	Engelbrecht	Johann	SAMF/UP
10	Fair	Andrew	Stellenbosch University
11	Frith	John	University of Cape Town
12	Gavhi-Molefe	Mpfareleni Rejoyce	African Institute for Mathematical Sciences
13	Govender	Kevin	South African Astronomical Observatory
14	Green	Barry	African Institute for Mathematical Sciences
15	Hahne	Fritz	STIAS
16	Hitchcock	Gavin	Stellenbosch University
17	Hofmeyr	Jan-Hendrik	Stellenbosch University
18	Holgate	David	University of the Western Cape
19	Jafftha	Jacob	University of Cape Town
20	Jordaan	Kerstin	University of Pretoria
21	Kalala	Franck	African Institute for Mathematical Sciences
22	Laurie	Henri	University of Cape Town



#	SURNAME	NAME	INSTITUTION
23	le Roux	Kate	University of Cape Town
24	Little	Francesca	University of Cape Town
25	Marshall	Delia	University of the Western Cape
26	McNamara	Caryn (Caz)	University of the Witwatersrand
27	Mpiyakhe	Zuki	ASSAf
28	Ndlovu	Mdu	Stellenbosch University
29	Nongxa	Loyiso	University of the Witwatersrand
30	Osano	Bob	University of Cape Town
31	Ouifki	Rachid	Stellenbosch University
32	Paterson	Mark	Rapporteur/Scribe
33	Potgieter	Marietjie	University of Pretoria
34	Prince	Robert	University of Cape Town
35	Reddy	Daya	University of Cape Town
36	Schafer	Marc	Rhodes University
37	Webb	Paul	Nelson Mandela Metropolitan University
38	Wittenberg	Martin	University of Cape Town

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$z = 0$



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